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Lagrangian with off-shell vertices and field redefinitions.

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Abstract

Meson exchange diagrams following from a lagrangian with off-shell meson-nucleon couplings are compared with those generated from conventional dynamics. The off-shell interactions can be transformed away with the help of a nucleon field redefinition. Contributions to to the NN- and $3N$-potentials and nonminimal contact e.m. meson-exchange currents are discussed, mostly for an important case of scalar meson exchange.

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I. INTRODUCTION

A nucleon-nucleon interaction is efficiently parametrized in terms of meson exchange diagrams. Apart from the largely model independent one pion exchange contribution, a small family of heavier mesons (ω, ρ, σ, sometimes also δ and η) effectively summing and representing the multipole pion exchanges and excitations of nucleon resonances in intermediate states is considered. The structure of the couplings of these (effective) mesons to nucleons is not very well known and is treated phenomenologically: vertices are usually taken in some simple form, neglecting effects due to possible transition of the nucleons off their mass shell, and coupling constants and coupling parameters are fitted to data.

Some relativistic calculations [1,2] have found that including off-shell extensions of these meson-nucleon vertices might give a more effective description. In particular, Stadler and Gross [1] have shown in a covariant spectator formalism that using a scalar-nucleon-nucleon (sNN) coupling with off-shell extension can give a reasonable triton binding energy (without explicit three-nucleon forces), and at the same time improve the fit to NN data as compared to that of similar model without off-shell coupling. Zimanyi and Moszkowski have shown that a lagrangian with a derivative (off-shell) sNN coupling improves the description of nuclear matter and finite nuclei in the relativistic mean-field approximation. And a number of authors have studied off-shell couplings using sidewise dispersion relations, which suggest that the off-shell behavior should be related to πN scattering and higher nucleon resonances [3,4].

Here, we would like to demonstrate how we can use nucleon field redefinition to translate the dynamical model of [1] into a model with nonlinear couplings with standard on-shell vertices. First, as an example, we demonstrate the nontrivial dynamical content of off-shell vertices using the well known σ-model. Then, in Sec. III, we consider the scalar-exchange part of the Stadler and Gross [1] model. We show that the off-shell coupling can be removed via a redefinition of the nucleon field and identify the nonstandard nonlinear strong and electromagnetic (e.m.) vertices. In leading order the difference between the model with off-shell coupling and the standard one is represented by triangle and bubble diagrams for the NN-interaction, scalar-scalar exchange three-nucleon potential, and a contact (seagull) meson exchange current. In Sec. IV we give explicit expressions for these contributions and argue that they should be estimated numerically. Finally, in Secs. V and VI we discuss electromagnetic interactions and some features of the nonrelativistic limits of these interactions. In the appendix we show that similar considerations also apply for pseudoscalar and vector exchanges.

II. σ-MODEL AS SIMPLE EXAMPLE

To illustrate the rich dynamical content of off-shell couplings, let us consider the simple example of a σ-model. The standard lagrangian of the linear σ- model is

$$\mathcal{L} = \mathcal{L}^{kin}_{N,0}(\psi) + \mathcal{L}^{kin}(\sigma,\pi) - g\bar{\psi}\Phi\psi - \frac{1}{2}m_\Phi^2\,\Phi^2 + V(\Phi^2)\,, \tag{2.1}$$

$$\mathcal{L}^{kin}_{N,0}(\psi) = \frac{i}{2}\bar{\psi}\gamma^\mu(\partial_\mu\psi) - \frac{i}{2}(\partial_\mu\bar{\psi})\gamma^\mu\psi\,, \tag{2.2}$$

$$\mathcal{L}^{kin}(\sigma,\pi) = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\,, \tag{2.3}$$

where $\Phi = \sigma + i\gamma^5\vec{\tau}\cdot\vec{\pi}$. The nonlinear σ-model results from replacing Φ by a nonlinear function of the pion field with constant norm $\Phi^2 = \Phi\Phi^* = f_\pi^2$. The common choice is

$$\Phi(\varphi) = f_\pi \exp(2i\gamma^5\varphi)\,, \tag{2.4}$$

with $\vec{\varphi} = \vec{\pi}/(2f_\pi)$ and $\varphi = \vec{\tau}\cdot\vec{\varphi}$, which leads to a lagrangian

$$\mathcal{L} = \mathcal{L}^{kin}_{N,0}(\psi) + \frac{1}{4}Tr\,\partial_\mu\Phi\partial^\mu\Phi^* - f_\pi g\bar{\psi}\exp(2i\gamma^5\varphi)\psi\,, \tag{2.5}$$

where the trace is taken in flavor space. In the lowest order in pion fields the second term gives the kinetic energy for massless pions, while higher orders generate complicated nonlinear pion self-interactions. The last term of (2.5) in the lowest order generates the nucleon mass $m = gf_\pi$, the first order in the pion field is the pseudoscalar πNN coupling, and higher orders represent multipion contact terms.

Alternatively, $\Phi(\varphi)$ can be taken in the form

$$\Phi(\varphi) = f_\pi\frac{1-\vec{\varphi}^2 + 2i\gamma^5\varphi}{1+\vec{\varphi}^2} = f_\pi + 2f_\pi\frac{-\vec{\varphi}^2 + i\gamma^5\varphi}{1+\vec{\varphi}^2}\,, \tag{2.6}$$

which yields

$$\mathcal{L} = \mathcal{L}^{kin}_{N,0}(\psi) + \frac{1}{2(1+\vec{\varphi}^2)^2}\partial_\mu\vec{\pi}\,\partial^\mu\vec{\pi} - f_\pi g\bar{\psi}\psi + \frac{2f_\pi g}{(1+\vec{\varphi}^2)}\bar{\psi}(\vec{\varphi}^2 - i\gamma^5\vec{\tau}\cdot\vec{\varphi}\,)\psi\,, \tag{2.7}$$

We can eliminate the higher order nonlinear terms from the πNN couplings by redefining the nucleon field as follows:

$$\psi(x) = \sqrt{1+\vec{\varphi}^2(x)}\,\psi'(x)\,, \tag{2.8}$$

which gives the equivalent lagrangian

$$\begin{aligned}\mathcal{L} = \mathcal{L}^{kin}_{N,0}(\psi') - f_\pi g\bar{\psi}'\psi' + \frac{1}{2(1+\vec{\varphi}^2)^2}\partial_\mu\vec{\pi}\,\partial^\mu\vec{\pi} + 2f_\pi g\bar{\psi}'(\vec{\varphi}^2 - i\gamma^5\vec{\tau}\cdot\vec{\varphi})\psi' \\ +\frac{1}{2}\bar{\psi}'\,\vec{\varphi}^2(i\gamma^\mu\partial_\mu\psi' - m\psi') - \frac{1}{2}(i(\partial_\mu\bar{\psi}') + m\bar{\psi}')\vec{\varphi}^2\,\psi'\,,\end{aligned} \tag{2.9}$$

Note that *all of the πNN interaction terms involving the coupling of three or more pions to a nucleon in the original lagrangian (2.7) have been replaced by terms which are only linear or quadratic in the pion field*, but which involve couplings to off-shell nucleons.

We now emphasize two points. The two lagrangians (2.7) and (2.9) are equivalent *only* if *all* of the nonlinear terms in the factor $f = 1/(1+\vec{\varphi}^2)$ are retained. If this factor is truncated to lowest order, $f \simeq 1 - \vec{\varphi}^2$, the lagrangians are *no longer equivalent*. The second point is that the lagrangian (2.9) is deceptively simple. In particular, at most two pions couple to a nucleon at any point. Still, it contains all the complexity of the nonlinear σ-model with all of its multipion contact terms. Naively, one might argue that the off-shell term is unimportant

in realistic nuclear applications, since it is nonzero only for off-shell nucleons. Indeed, in the lowest semiclassical order of two meson exchange it does not contribute. But as momentum loops are included the off-shell vertices give results compatible with original nonlinear σ-model. Iterating the off-shell vertices of (2.9) along the same nucleon line immediately generates multipion contact terms with any number of pions, since the off-shell factors ($\hat{p} - m$ in momentum space) of the vertex cancel the attached nucleon propagators.

However, it might be nontrivial to define the effective theory based on the lagrangian (2.9). This is because the nucleon field has complicated transformation properties and one has to be careful in defining the regularization so that the symmetry is preserved and the theory is equivalent to the usual nonlinear σ-model. We are interested in a more phenomenological approach in which only a limited set of Feynman diagrams (regularized by ad hoc hadronic form factors) is used in a kernel of a dynamical equation. In this case, the dynamics defined by (2.9) is no longer fully equivalent to the σ-model, but it is still clearly distinct from the standard prescriptions employing the linearized form of the πNN interaction.

We now turn to a discussion of off-shell couplings associated with the exchange of scalar particles.

III. FIELD REDEFINITIONS FOR OFF-SHELL SCALAR COUPLING

Let us now consider the scalar coupling with the off-shell extension which plays an important role in the dynamical model discussed in Ref. [1]. We will first show how the off-shell part of the sNN coupling can be removed with the help of the nucleon field redefinition. The field redefinition generates the nonlinear scalar-nucleon and photon-scalar-nucleon vertices. In the next sections, we present the contributions of these vertices to the NN potential, the $3N$ potential and the e.m. exchange currents in the leading order beyond conventional results.

The part of lagrangian relevant to our discussion is

$$\mathcal{L} = \mathcal{L}_B^{kin} + \mathcal{L}_{\gamma ss} + \mathcal{L}_N^{kin}(\psi) + \mathcal{L}_{sNN}(\psi) + \mathcal{L}_{\gamma NN}(\psi) + \mathcal{L}_{\gamma NNs}(\psi)\,, \tag{3.1}$$

$$\mathcal{L}_N^{kin}(\psi) = \frac{i}{2}\bar{\psi}\gamma^\mu(\partial_\mu\psi) - \frac{i}{2}(\partial_\mu\bar{\psi})\gamma^\mu\psi - m\bar{\psi}\psi\,, \tag{3.2}$$

$$\mathcal{L}_{sNN}(\psi) = g_s\bar{\psi}\Phi_s\psi + \frac{a_s}{2}\bar{\psi}\Phi_s(i\gamma_\mu\partial^\mu\psi - m\psi) - \frac{a_s}{2}(i(\partial^\mu\bar{\psi})\gamma_\mu + m\bar{\psi})\Phi_s\psi\,, \tag{3.3}$$

$$\mathcal{L}_{\gamma NN}(\psi) = \bar{\psi}\Lambda^\mu\psi\, A_\mu \quad , \; \Lambda^\mu = \Lambda_0^\mu + \Delta\Lambda^\mu \tag{3.4}$$

$$\mathcal{L}_{\gamma NNs}(\psi) = \frac{a_s}{2}\bar{\psi}\left\{\Lambda_0^\mu\,, \Phi_s\right\}\psi\, A_\mu\;, \tag{3.5}$$

where $\mathcal{L}_B^{kin}$ includes all of the kinetic terms for the bosons (scalars and photons), $\mathcal{L}_N^{kin}(\psi)$ is the nucleon kinetic lagrangian with the mass term, and $\mathcal{L}_{\gamma ss}$ is a photon-scalar vertex.

The function Φ_s contains an isospin matrix for the mesons with nonzero isospin (i.e., $\Phi_s = \vec{\tau}\cdot\vec{\Phi}_s$). The parameter a_s in the off-shell part of the sNN vertex is related to the parameter ν_s of [1] through $a_s = \nu_s g_s/m$ and the sNN vertex function in the momentum space reads

$$\tilde{\Gamma}_s(p', p) = 1 + \frac{\nu_s}{2m}\left(\hat{p}' + \hat{p} - 2m\right)\,, \tag{3.6}$$

In (3.4) we have separated a minimal part of the γNN vertex

$$\Lambda_0^\mu = \frac{e}{2}(1+\tau^3)\,\gamma^\mu\,, \tag{3.7}$$

with the proton charge $e > 0$, from the remaining purely transverse one $\Delta\Lambda^\mu$. The minimal contact (seagull) interaction is contained in $\mathcal{L}_{\gamma NNs}(\psi)$. It is obtained by minimal substitution of the derivatives of the nucleon field in (3.3)

$$i\gamma^\mu \partial_\mu \psi \to \Lambda_0^\mu \psi\,, \tag{3.8}$$
$$-(i\partial_\mu \bar{\psi}\gamma^\mu) \to \bar{\psi}\Lambda_0^\mu\,. \tag{3.9}$$

The nonminimal part of the photon-nucleon coupling $\Delta\Lambda^\mu$ has been given in a framework of effective meson-nucleon theories by Gross and Riska [7]. In simplified models, Λ_0^μ and $\Delta\Lambda^\mu$ are often taken as the Dirac and Pauli parts of the γNN vertex, proportional to F_1 and F_2, respectively.

The similarity of (3.2) and (3.3) suggests that the off-shell vertex can be removed by the nucleon field redefinition [2]

$$\psi(x) = F_s(\Phi_s(x))\,\psi'(x)\,, \tag{3.10}$$
$$\bar{\psi}(x) = \bar{\psi}'(x) F_s(\Phi_s(x)), \tag{3.11}$$
$$F_s(x) = 1/\sqrt{1 + a_s\Phi_s(x)}\,. \tag{3.12}$$

In terms of the new field ψ' the lagrangian (3.1) reads

$$\begin{aligned}
\mathcal{L} = {} & \mathcal{L}_s^{kin} + \mathcal{L}_N^{kin}(\psi') + \mathcal{L}_{\gamma ss} \\
& + g_s\bar{\psi}'\frac{\Phi_s}{1+a_s\Phi_s}\psi' + \frac{i}{2}\bar{\psi}'\gamma^\mu\left(F_s^{-1}(\partial_\mu F_s) - (\partial_\mu F_s)F_s^{-1}\right)\,\psi' \\
& + \bar{\psi}' F_s\,\Lambda^\mu\,F_s\psi'\,A_\mu + \frac{a_s}{2}\bar{\psi}' F_s\,\left\{\Lambda_0^\mu\,,\Phi_s\right\}\,F_s\psi'\,A_\mu\;.
\end{aligned} \tag{3.13}$$

As in the previous section, the off-shell meson-nucleon vertex can be transformed away in favor of complicated nonlinear contact interactions. The leading term of the nonlinear scalar-nucleon term is the conventional scalar-nucleon vertex, the terms with derivatives of the scalar field $(\partial_\mu F_s)$ contribute only for the scalar with nonzero isospin.

While the off-shell interaction can be easily included in covariant dynamical equations for NN and $3N$ systems, one cannot take the nonlinear vertices to all orders, and hence in practice the two forms of the lagrangian are not equivalent (although they are in principle). In an energy region where the effective meson-nucleon description is valid, the importance of the multimeson exchanges and/or exchanges of heavy mesons decreases with increasing summed mass of exchanged mesons. In particular, since the mass of the effective scalar meson is typically $m_s \approx 500$ MeV, it might be interesting to consider the effects of nonlinearities up to second order in the scalar field, and see if the differences between a truncated version of (3.13) and the orignial (3.1) can be explained mostly by the second order terms.

Since the original lagrangian (3.1) depends on the off-shell parameter ν (or a), so also will physical observables, like phase shifts and the $3N$ binding energy (parameters of the model [1] are fine tuned to leave the deuteron binding energy unchanged). However, the

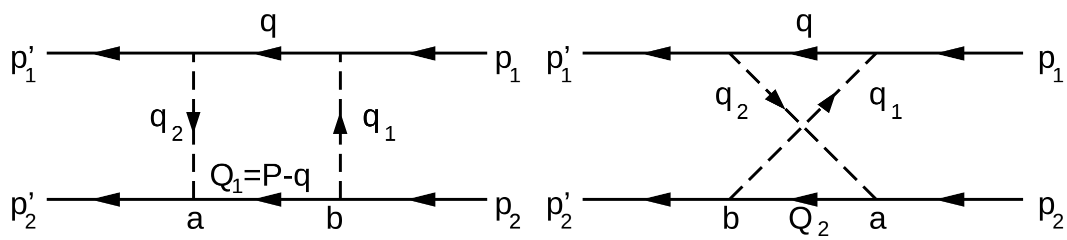

FIG. 1. Box and crossed box contributions to the two-meson exchange NN potential.

leading order one meson exchange interaction does not depend on ν, at least for external legs on-shell. But nucleon interactions with two exchanged mesons, or interactions with external fields with at least one meson simultaneously exchanged, are already dependent on the choice of ν. We present the corresponding ν-dependent operators below, first for the two-scalar exchange NN and $3N$ potentials and then for the one-scalar exchange e.m. current.

IV. TWO-SCALAR EXCHANGE AND NUCLEAR INTERACTION

Since the local field redefinition does not change the physical S-matrix elements, the NN-interaction generated by the lagrangian (3.13) with nonlinear sNN couplings should give the same result as the lagrangian (3.1) where the nonlinear couplings are replaced by off-shell couplings. It is easy to see that the leading, second order contributions due to a single scalar exchange are the same in both frameworks. In this section we show that this is also true of the fourth order, two scalar exchange terms. The demonstration illustrates explicitly how the off-shell couplings generate higher order contact terms.

Two scalar exchange contributions with off-shell couplings are represented by the box and crossed box contributions of Fig. 1. Using (3.6) the corresponding amplitudes are

$$\begin{aligned}\mathcal{M}_{box} = i g_s^4 \int \frac{d^4 q}{(2\pi)^4} D(q_1) D(q_2)\, \bar{u}(p_1')\tilde{\Gamma}_s^a(p_1', q) G(q) \tilde{\Gamma}_s^b(q, p_1) u(p_1) \\ \times \bar{u}(p_2')\tilde{\Gamma}_s^a(p_2', Q_1) G(Q_1) \tilde{\Gamma}_s^b(Q_1, p_2) u(p_2)\,, \qquad (4.1)\end{aligned}$$

$$\begin{aligned}\mathcal{M}_{cross} = i g_s^4 \int \frac{d^4 q}{(2\pi)^4} D(q_1) D(q_2)\, \bar{u}(p_1')\tilde{\Gamma}_s^a(p_1', q) G(q) \tilde{\Gamma}_s^b(q, p_1) u(p_1) \\ \times \bar{u}(p_2')\tilde{\Gamma}_s^b(p_2', Q_2) G(Q_2) \tilde{\Gamma}_s^a(Q_2, p_2) u(p_2)\,, \qquad (4.2)\end{aligned}$$

where $G(p) = 1/(m - \hat{p} - i\epsilon)$ is the nucleon propagator, $D(q) = 1/(m_s^2 - q^2 - i\epsilon)$ is the scalar propagator, and the momenta are defined in Fig. 1.

The sum of these amplitudes should be equal to the sum of the box and crossed-box diagrams with the on-shell vertex $\Gamma_s = 1$ and the triangle and bubble diagrams of Fig. 2. These triangle and bubble diagrams are generated from the quadratic contact vertex

$$\mathcal{L}_{ssNN}(\psi') = -\frac{g_s^2 \nu_s}{m} \bar{\psi}' \Phi_s^2 \psi' + \frac{g_s^2 \nu_s^2}{4m^2} \bar{\psi}' \gamma^\mu \vec{\tau} \cdot \vec{\Phi}_s \times \partial_\mu \vec{\Phi}_s \psi'\,, \qquad (4.3)$$

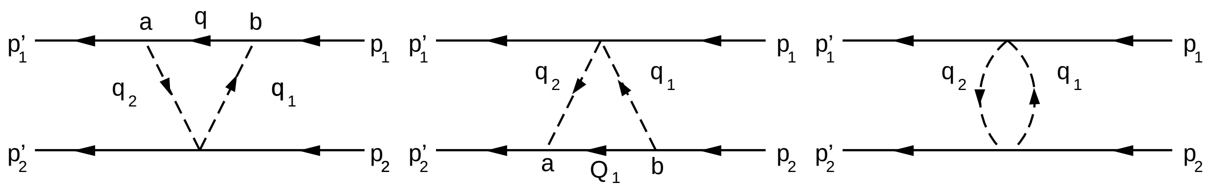

FIG. 2. Triangle and bubble contributions to the two-scalar exchange NN potential generated by the contact interactions in the transformed lagrangian.

which follows from the Taylor decomposition of the nonlinear vertex in (3.13). In the momentum representation the quadratic contact vertex is

$$\Gamma^{ab}(q_2, q_1) = -2g_s^2 \frac{\nu_s}{m} \left(\delta^{ab} + i\epsilon^{abc}\tau^c \frac{\nu_s}{8m}(\hat{q}_1 + \hat{q}_2) \right) , \tag{4.4}$$

where a and b are isospin indices associated with the incoming $\Phi^b(q_1)$ and outgoing $\Phi^a(q_2)$ scalar fields. The amplitudes corresponding to these diagrams in Fig. 2 are

$$\begin{aligned}\mathcal{M}_{tri} = -ig_s^4 \int \frac{d^4q}{(2\pi)^4} D(q_1)D(q_2)\, 2\frac{\nu_s}{m} \Big\{ & \delta^{aa} \left[\bar{u}(p_1')G(q)u(p_1)\bar{u}(p_2')u(p_2) \right. \\ & \left. + \bar{u}(p_1')u(p_1)\bar{u}(p_2')G(Q_1)u(p_2) \right] + \frac{\nu_s}{2m} \left[\bar{u}(p_1')G(q)\tau^a u(p_1)\bar{u}(p_2')\hat{Q}\tau^a u(p_2) \right. \\ & \left. -\bar{u}(p_1')\hat{Q}\tau^a u(p_1)\bar{u}(p_2')G(Q_1)\tau^a u(p_2) \right] \Big\} ,\end{aligned} \tag{4.5}$$

$$\begin{aligned}\mathcal{M}_{bub} = ig_s^4 \int \frac{d^4q}{(2\pi)^4} D(q_1)D(q_2)\, 2\frac{\nu_s^2}{m^2} \Big\{ & \delta^{aa}\bar{u}(p_1')u(p_1)\bar{u}(p_2')u(p_2) \\ & + \frac{\nu_s^2}{8m^2}\bar{u}(p_1')\hat{Q}\tau^a u(p_1)\bar{u}(p_2')\hat{Q}\tau^a u(p_2) \Big\} .\end{aligned} \tag{4.6}$$

where we have introduced $Q = (q_1 + q_2)/2$. The equivalence of $\mathcal{M}_{box} + \mathcal{M}_{cross}$ to the sum of conventional box and crossed-box diagrams and $\mathcal{M}_{tri} + \mathcal{M}_{bub}$ given above follows from simple algebra. The substitution $q \to -q + p_1' + p_1$ in the integral, which leads to replacements $q_1, q_2, Q_1, Q_2, Q \to -q_2, -q_1, Q_2, Q_1, -Q$, is useful in the proof.

If the scalar field has a zero isospin, $\delta^{ab}, \delta^{aa} \to 1$ and terms with other isospin structures disappear. The terms containing $\hat{Q}$ in Eqns. (4.5) and (4.6) are suppressed by extra powers of $1/m$. Hence, at least in the leading order, the ν_s-dependent two-scalar exchange contributions have very simple structure. The ν_s-dependent two-scalar exchange NN potentials follow from (4.5) and (4.6) by a straightforward nonrelativistic reduction of the vertices, in particular $\bar{u}(p')u(p) \to 1$.

The triangle and bubble diagram contributions to the NN potential in the leading order in $1/m$ were recently constructed by Rijken and Stoks [6] for pions and σ-mesons, where only positive-energy nucleons were considered in intermediate states. It would be interesting to check whether the contributions derived above could account for a sizable part of dynamical difference between a model with the standard sNN coupling and a model with its off-shell extension. To this end one should fix all other meson parameters and fit the conventional

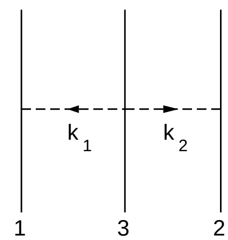

FIG. 3. Contribution to the trinucleon potential generated by the contact interactions in the transformed lagrangian. Two other diagrams with cyclic permutation of nucleons are not shown.

model to the NN data. Then one could compare the effects of adding either an off-shell vertex or the triangle and bubble interactions to the potential.

The two-scalar exchange also contributes to the three-nucleon potential shown on Fig. 3. In particular, from the ν-dependent quadratic vertex (4.4) one gets

$$\begin{aligned} V_{3N} = 2\frac{\nu_s}{m}\, g_s^4\, D(k_1)D(k_2)\, \bar{u}(p_1')\tau^a u(p_1)\, \bar{u}(p_2')\tau^b u(p_2) \\ \bar{u}(p_3')\left[\delta^{ab} + i\epsilon^{cab}\tau^c \frac{\nu_s}{8m}(\hat{k_1} - \hat{k_2})\right] u(p_3) + acycl\,. \end{aligned} \tag{4.7}$$

The potential simplifies when only lowest order in v/c is retained, which means replacing all the vertices by unity. This gives a very simple, central three-nucleon potential, which is attractive for $\nu_s < 0$, in agreement with the increased binding for negative ν_s observed in [1]. Together with the triangle contributions to the NN interaction discussed above, this three-nucleon potential should account for part of the large effect of the off-shell scalar coupling on the triton binding energy [1]. It might be interesting to check this numerically and to compare the importance of the three-nucleon force to variations of the NN-interaction due to additional two-scalar exchanges.

V. ELECTROMAGNETIC INTERACTION

Since the e.m. part of the transformed lagrangian (3.13) also contains complicated nonlinear multimeson interactions, the comparison of the e.m. observables calculated using nonlinear models or models with off-shell couplings requires some care. However, in the spirit of the previous section one can try to explain a part of the difference by estimating the leading order effects, which for the e.m. interaction are nonminimal single-scalar exchange e.m. currents. Making the Taylor decomposition of the e.m. part of (3.13) with the help of $F_s \simeq 1 - a_s\Phi_s/2$ we get

$$\begin{aligned} \mathcal{L}_{e.m.} &= \bar{\psi}' F_s\, \Lambda^\mu\, F_s\psi'\, A_\mu + \frac{a_s}{2}\bar{\psi}' F_s\, \{\Lambda_0^\mu\,, \Phi_s\}\, F_s\psi'\, A_\mu \\ &\simeq \bar{\psi}'\Lambda^\mu\psi'\, A_\mu - \frac{a_s}{2}\bar{\psi}'\, \{\Delta\Lambda^\mu\,, \Phi_s\}\, \psi'\, A_\mu\,. \end{aligned} \tag{5.1}$$

The minimal part of the contact γNNs vertex proportional to Λ_0^μ disappears, since the lagrangian (3.13) does not contain derivatives of the scalar field in the linear sNN vertex.

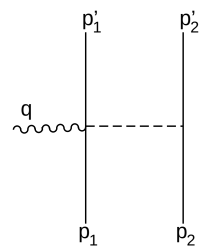

FIG. 4. Contribution to the electromagnetic meson-exchange current generated by the contact interactions in the transformed lagrangian. Only the diagram with the contact vertex attached to the first nucleon is shown.

Instead, a nonminimal, purely transverse contact interaction appears. It is, of course, not clear that the lagrangian with off-shell coupling from which we start should have *a minimal* contact e.m. coupling. But, in any case, the field definition (3.12) transforms the nuclear electromagnetic current (3.4) into an interaction current, which to lowest order in the scalar field is

$$\delta\mathcal{L}_{\gamma NNs} = -\frac{a_s}{2}\bar{\psi}'\left\{\Lambda^\mu\,,\Phi_s\right\}\,\psi'\,A_\mu\,. \tag{5.2}$$

This must be added to the transformation of whatever other contact interaction replaces (3.5) in the original lagrangian.

One can easily write down the interaction current $\delta j^\mu_{s,con}(q)$, corresponding to (5.2) and given in Fig. 4

$$\begin{aligned}\delta j^\mu_{s,con}(q) = \frac{g_s a_s}{2}\,& D_s(p_2'-p_2)\,\bar{u}(p_1')\left(\tau^a\Lambda^\mu(p_1+q,p_1)+\Lambda^\mu(p_1',p_1'-q)\tau^a\right)u(p_1)\\ &\times\bar{u}(p_2')\tau^a u(p_2)\,,\end{aligned} \tag{5.3}$$

where for a scalar meson with isospin $I=0$ one has to replace $\tau^a \to 1$. This current should account for some part of the difference between the e.m. observables, e.g., deuteron form factors, calculated in the conventional framework and one with the scalar off-shell coupling.

In the lowest order in v/c, this term gives the following contribution to the charge density

$$\delta\rho_{s,con} \simeq \frac{\nu_s g_s^2}{2m}\left\{\hat{e}_1, \vec{\tau}_1\cdot\vec{\tau}_2\right\}\,D_s(p_2'-p_2)\,, \tag{5.4}$$

where $\hat{e}_1$ is the charge of the first nucleon as an operator in isospin space. This term has a structure which is very similar to the retardation contribution from sigma exchange estimated in Ref. [8], which was found to give a nonnegligible contribution to the deuteron and trinucleon form factors. We therefore expect (5.4) also to give a nonnegligible effect.

VI. OFF-SHELL EFFECTS AND NONRELATIVISTIC EXPANSIONS

It is instructive to compare our results for off-shell scalar exchange to results which have been previously obtained from the study of off-shell pion exchanges. These studies have

been carried out in the framework of conventional perturbative expansions of boson-nucleon vertices in powers of v/c, time-ordered diagrams for potentials and currents, and wave functions obtained from Schrödinger-like equations. In this framework, the nucleons are on their mass shells, but energy is not conserved at the vertices and therefore the potential and the current operators do not commute with the hamiltonian. The definition of these operators off-shell is not unique, is subject to ambiguities, and varies for different methods. For pion exchange [9–11] it has been shown [5,9] that all the results of various methods are, to leading relativistic order, covered by a generic formula

$$A(\tilde{\mu}) = i\tilde{\mu}\,[A_{n.r.}, U] + \Delta A(\mu)\,, \tag{6.1}$$

where $A_{n.r.}$ is a corresponding nonrelativistic operator, U is hermitian interaction-dependent operator $\sim (v/c)^2$, and a parameter $\tilde{\mu}$ depends on the particular method used [see below (6.6)] and the PS-PV mixing parameter μ, defined in (A.5) with $\nu_{ps} \to \mu$ for pions. The last term on the r.h.s. of (6.1), $\Delta A(\mu)$, follows from the explicit dependence of the *underlying lagrangian* on the mixing parameter μ, at the one-pion exchange level it is just the nonminimal contact exchange current. If the chiral lagrangian is employed $\Delta A(\mu) = 0$. The commutator terms generate *additional* exchange currents, a contribution to *one-pion-exchange* potential, and, if two-pion exchanges are included, also two-pion-exchange NN and $3N$ potentials.

In this section we show, that for scalars there is no additional unitary freedom and terms analogous to the commutator in (6.1) do not appear. Therefore, the one-scalar-exchange potential does not depend on the off-shell parameter ν and the explicitly ν-dependent potentials and currents derived in this paper correspond to the last term in (6.1) and they appear because the original lagrangian does depend on ν.

Let us first recall the results for the pseudoscalar mesons [9–11]. From the lagrangian with mixture of the PS and PV couplings one gets

$$i\partial_t\psi = \left\{\vec{\alpha}\cdot\vec{p} + \gamma^0 m + ig(1-\mu)\gamma^0\gamma^5\Phi - \frac{\mu g}{2m}(\vec{\sigma}\cdot\vec{\nabla}\Phi) - \frac{\mu g}{2m}\gamma^5(\partial_t\Phi)\right\}\psi = H\psi\,. \tag{6.2}$$

Removing the odd operators by a standard FW procedure (see e.g., chapter 4 of [14]), one gets

$$\begin{aligned} H^{int}_{FW} = -\frac{g}{2m}\Big(& (\vec{\sigma}\cdot\vec{\nabla}\Phi) - \frac{1}{4m^2}\{\vec{p}^{\,2}, (\vec{\sigma}\cdot\vec{\nabla}\Phi)\} + (1+\mu+c(1-\mu))\frac{1}{4m}\gamma^0\{\vec{\sigma}\cdot\vec{p}, (\partial_t\Phi)\} \\ & + (\mu + c(1-\mu)]\frac{i}{8m^2}\{\vec{\sigma}\cdot\vec{p}, [\vec{p}^{\,2}, \Phi]\}\Big)\,, \end{aligned} \tag{6.3}$$

where c, defined as in [10,11], is so-called Barnhill parameter [12] which determines in which order are the odd terms eliminated. In the momentum space, the vertex function for i-th nucleon interacting with pion derived from (6.3) is

$$\begin{aligned} \Gamma(p'_i, p_i) = \frac{i}{2m}\Big[& (\vec{\sigma}_i\cdot\vec{q}_i)\left(1 + \frac{\vec{p}_i^{\,\prime\,2} + \vec{p}_i^{\,2}}{4m^2}\right) \\ & - \vec{\sigma}_i\cdot(\vec{p}_i^{\,\prime} + \vec{p}_i)\,\frac{1}{4m}\Big(\Delta E_i - (1+\mu+c(1-\mu))(\Delta E_i - q_{0,i})\Big)\Big]\,, \end{aligned} \tag{6.4}$$

where $\Delta E_i = E'_i - E_i$ is the energy difference for final and initial on-mass-shell nucleon and $q_{0,i}$ is an energy of the absorbed meson. For pions, the unitary freedom appears because one

can extend the vertex off-energy- shell, i.e., set $\Delta E_i \neq q_{0,i}$. In fact, it has been shown [5,13] that various ways to construct two-nucleon operators effectively replace

$$\Delta E_i - q_{0,i} = \beta(E_1' + E_2' - E_1 - E_2) = \beta(E' - E)\,, \tag{6.5}$$

where β is an arbitrary parameter, introduced in [13], and for most techniques in use $\beta = 1/2$. The unitary parameter $\tilde{\mu}$ from (6.1) is given by

$$1 + \tilde{\mu} = 2\beta\left[\mu + 1 + c(1-\mu)\right]\,, \tag{6.6}$$

and it combines dependence on the off-shell parameter μ with dependence on c and β, which parametrize all of the various ways of doing FW transformations and constructing interaction-dependent operators. For any value of $\tilde{\mu}$ one gets a hermitian hamiltonian and a description conforming with approximate Lorentz and gauge invariance.

Let us now consider the case of scalar mesons. From the lagrangian (3.1) we obtain the equation for the time dependence of the nucleon field

$$\begin{aligned} i\partial_t\psi &= \left\{\vec{\alpha}\cdot\vec{p} + \gamma^0 m - g(1-\nu)\gamma^0\Phi + \frac{a}{2}\left\{\vec{\alpha}\cdot\vec{p},\,\Phi\right\} - \frac{a}{2}i(\partial_t\Phi) - a\Phi i\partial_t\right\}\psi \\ &\simeq \left\{\vec{\alpha}\cdot\vec{p} + \gamma^0 m - g\gamma^0\Phi + \frac{a}{2}\left[\vec{\alpha}\cdot\vec{p},\,\Phi\right] - \frac{a}{2}i(\partial_t\Phi)\right\}\psi = H\,\psi\,, \end{aligned} \tag{6.7}$$

where the second form follows from approximation

$$i\partial_t\psi \simeq (\vec{\alpha}\cdot\vec{p} + \gamma^0 m)\psi \tag{6.8}$$

on r.h.s. (recall that only terms linear in Φ are retained). Removing the odd operators, we obtain from the second form

$$H_{FW}^{int} = -g\gamma^0\left[\Phi - \frac{1}{8m^2}\left\{\vec{\sigma}\cdot\vec{p},\left\{\vec{\sigma}\cdot\vec{p},\Phi\right\}\right\}\right] + \frac{\nu g}{2m}\left(-i(\partial_t\Phi) + \gamma^0\left[\frac{\vec{p}^{\,2}}{2m},\Phi\right]\right). \tag{6.9}$$

Note that, unlike for pions, there is no Barnhill freedom at the order considered. The reason is that the interaction-dependent odd terms of the untransformed hamiltonian (6.7) are of the order $\sim 1/m$ and hence the corresponding unitary transformation would generate the contributions $\sim 1/m^3$, while we keep only the terms up to $\sim 1/m^2$. In momentum space, the hamiltonian (6.9) generates the sNN vertex for i-th nucleon

$$\Gamma(p_i', p_i) = \left[1 - \frac{1}{8m^2}\left((\vec{p}_i^{\,\prime} + \vec{p}_i)^2 + 2i\vec{\sigma}_i\cdot(\vec{p}_i^{\,\prime}\times\vec{p}_i)\right) - \frac{\nu}{2m}(\Delta E_i - q_{0,i})\right]. \tag{6.10}$$

For our scalar exchange this off-energy shell extension (6.5) is not allowed. The point is, that if the last ν-dependent term is present in (6.10), the vertex and the one-scalar exchange potential derived from it are not hermitian, i.e., $\Gamma(p_i', p_i) \neq \Gamma(p_i, p_i')^*$. It is clear already from (6.7), since the hamiltonian defined in (6.7) is hermitian only if

$$i(\partial_t\Phi) = \gamma^0\left[\frac{\vec{p}^{\,2}}{2m},\Phi\right]\,, \tag{6.11}$$

that is only if energy is conserved at the vertex.

We could attempt to re-write the eq. (6.7) with the hamiltonian which is equivalent to the old one on energy shell and which is hermitian

$$i\partial_t\psi = \left\{\vec{\alpha}\cdot\vec{p} + \gamma^0 m - g(1-\nu)\gamma^0\Phi + \frac{a}{2}\left\{\vec{\alpha}\cdot\vec{p},\, \Phi\right\} - \frac{a}{2}\left\{i\partial_t\,,\Phi\right\}\right\}\psi = H'\psi\,, \tag{6.12}$$

but with the help of (6.8) one easily sees that to first order in Φ, the hamiltonian H' is equivalent to the standard ν-independent one

$$H' \simeq \vec{\alpha}\cdot\vec{p} + \gamma^0 m - g\gamma^0\Phi\,, \tag{6.13}$$

and it gives only the standard part of (6.10). This can also be checked by straightforward FW transformation of (6.12). Therefore, we conclude that there is no consistent way to continue the sNN vertex off-energy shell. Consistent operators are obtained only if one puts $\beta = 0$, i.e., $\tilde{\mu} = -1$ in (6.5) and (6.6), respectively, as in the S-matrix approach [16]. Then, the nucleon potentials and exchange currents are identified with the straightforward Taylor expansion in powers of of $v/c \sim p/m$ of the corresponding Feynman amplitudes. This identification is made in previous sections. In particular, only the standard ν-independent part of the sNN vertex contributes to one-scalar-exchange potential and this potential does not depend on ν even when the relativistic corrections are included and the potential is considered off-energy-shell.

VII. CONCLUSIONS

The lagrangian with the off-shell vertices can be conveniently related to the more conventional one by means of the nucleon field redefinition. The transformed lagrangian contains simpler bNN vertices, but has complicated contact multimeson terms. One can also start from a conventional lagrangian and introduce off-shell couplings and some contact terms. Notice that this way one never gets the coupling with off-shell extension both before and after meson is emitted.

The nonlinear interactions generate triangle and bubble diagrams for the NN interaction, complicated three-nucleon interactions and meson exchange currents. We list leading order contributions to these operators.

For the scalar, pseudoscalar and isoscalar vector mesons one can completely transform away the interaction terms with derivatives of the nucleon field, for the isovector vector mesons this is possible only at the lowest order, linear in the vector field.

Since in the low and intermediate energy region the importance of nucleon operators is increasing with decreasing exchanged meson mass, it would be interesting to numerically estimate the effect of these lowest order operators, since they might represent a large part of the difference between a model with off-shell coupling and one with conventional vertices.

Unlike for the pion case, for scalars there is no additional unitary freedom dependent on the off-shell parameter ν in the framework of a hamiltonian formalism with a v/c expansion.

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APPENDIX A: PSEUDOSCALAR AND VECTOR EXCHANGES

In this appendix we demonstrate how the removal of the off-shell [1] coupling works for pseudoscalar and vector mesons. In a generic form the lagrangian with an off-shell coupling reads

$$\mathcal{L} = \mathcal{L}_B^{kin} + \mathcal{L}_{\gamma bb} + \mathcal{L}_N^{kin}(\psi) + \mathcal{L}_{bNN}(\psi) + \mathcal{L}_{\gamma NN}(\psi) + \mathcal{L}_{\gamma NNb}(\psi)\,, \tag{A.1}$$

$$\mathcal{L}_{bNN}(\psi) = g\bar{\psi}\Gamma\Phi\psi + \frac{a}{2}\bar{\psi}(\Gamma\Phi)(i\gamma_\mu\partial^\mu\psi - m\psi) - \frac{a}{2}(i(\partial^\mu\bar{\psi})\gamma_\mu + m\bar{\psi})(\Gamma\Phi)\psi\,, \tag{A.2}$$

$$\mathcal{L}_{\gamma NNb}(\psi) = \frac{a}{2}\bar{\psi}\left\{\Lambda_0^\mu\,,\Gamma\Phi\right\}\psi\, A_\mu\ , \tag{A.3}$$

where we use $b = s, v, ps$ to denote different mesons and $B = b, \gamma$ stands for both mesons and photons. The bNN vertex Γ contains some Dirac matrix structure, and it can carry a Lorentz index for spin 1 mesons and an isospin index for isospin 1 mesons. In more detail, we take for scalar mesons $\Gamma_s = 1$, for pseudoscalar mesons $\Gamma_{ps} = -i\gamma^5$ and for vector mesons $\Gamma_v^\mu = \gamma^\mu$. The vBB vertex could also contain an anomalous tensor coupling, but the off-shell extension of this part has not been considered in [1] and we omit the anomalous coupling here for the sake of simplicity . The bNN vertex function in momentum space reads

$$\tilde{\Gamma}(p',p) = \Gamma(p',p) + \frac{\nu}{2m}\left((\hat{p}' - m)\Gamma(p',p) + \Gamma(p',p)(\hat{p} - m)\right)\,, \tag{A.4}$$

For the pseudoscalar mesons the off-shell couplings (A.4) is just the usual pseudovector vertex,

$$\tilde{\Gamma}_{ps}(p',p) = -i\left((1-\nu_{ps})\gamma^5 + \frac{\nu_{ps}}{2m}(\hat{p}' - \hat{p})\gamma^5\right)\,, \tag{A.5}$$

which also follows from adding the total derivative to (A.1). For pions ν_{ps} is usually named μ. This is possible since $\{\gamma^\mu, \gamma^5\} = 0$ and it completely removes the derivatives of the nucleon fields from the interaction terms, introducing instead terms with derivatives of the meson field, i.e., replacing the difference of nucleon momenta by the meson momentum $q = p' - p$. Hence, the field redefinition removing this part of vertex should be equivalent to the usual chiral rotation which dials between PS and PV couplings. Nevertheless, let us

recover these results proceeding in a same way as used for scalar and vector mesons, though for pseudoscalar mesons the procedure appears somewhat artificial.

To transform away the off-shell bNN, we redefine the nucleon field with the help of the function $F = F(\Gamma\Phi)$, obeying

$$\gamma^0 F^*(\Gamma\Phi)\gamma^0 = F(\Gamma\Phi)\,, \tag{A.6}$$

so that

$$\psi(x) = F(\Gamma\Phi(x))\,\psi'(x)\,, \tag{A.7}$$

$$\bar{\psi}(x) = \bar{\psi}'(x)F(\Gamma\Phi(x))\;. \tag{A.8}$$

In terms of ψ'

$$\begin{aligned}\mathcal{L} = {} & \frac{i}{2}\bar{\psi}'F(1+a\Gamma\Phi)\gamma^\mu F(\partial_\mu\psi') - \frac{i}{2}(\partial_\mu\bar{\psi}')F\gamma^\mu(1+a\Gamma\Phi)F\psi' - m\bar{\psi}'F(1+a\Gamma\Phi)F\psi' \\ & + g\bar{\psi}'F^2(\Gamma\Phi)\psi' + \frac{i}{2}\bar{\psi}'F(1+a\Gamma\Phi)\gamma^\mu(\partial_\mu F)\psi' - \frac{i}{2}\bar{\psi}'(\partial_\mu F)\gamma^\mu F(1+a\Gamma\Phi)\psi' \\ & + \bar{\psi}'F\,\Lambda^\mu\,F\psi'\,A_\mu + \frac{a}{2}\bar{\psi}'F\,\left\{\Lambda_0^\mu\,,\Gamma\Phi\right\}\,F\psi'\,A_\mu\,.\end{aligned} \tag{A.9}$$

Requiring that the first line in (A.9) equals the nucleon kinetic lagrangian $\mathcal{L}_N^{kin}(\psi')$ implies

$$\gamma^\mu = F(1+a\Gamma\Phi)\gamma^\mu F = F\gamma^\mu(1+a\Gamma\Phi)F\,, \tag{A.10}$$

which can be satisfied only if $[\gamma^\mu\,,\Gamma] = 0$. However, this does not mean that it is possible to transform away interactions with derivatives of nucleon fields only in this case: the offending terms might be eliminated by adding a total derivative. Indeed, let us denote

$$\Gamma_1^\mu = F(1+a\Gamma\Phi)\gamma^\mu F\,, \tag{A.11}$$

$$\Gamma_2^\mu = F\gamma^\mu(1+a\Gamma\Phi)F\,, \tag{A.12}$$

and add to the lagrangian (A.9) a total derivative

$$\frac{i}{2}\partial_\mu\left[\bar{\psi}'\left(f\Gamma_1^\mu - \Gamma_2^\mu f\right)\psi'\right], \tag{A.13}$$

where $f = f(\Gamma\Phi)$ is to be determined to allow reduction of all terms with derivatives of nucleon fields to $\mathcal{L}_N^{kin}(\psi')$. These terms now equal

$$\frac{i}{2}\bar{\psi}'\left((f+1)\Gamma_1^\mu - \Gamma_2^\mu f\right)(\partial_\mu\psi') - \frac{i}{2}(\partial_\mu\bar{\psi}')\left(\Gamma_2^\mu(f+1) - \Gamma_1^\mu f\right)\psi'\,. \tag{A.14}$$

Requiring that

$$\gamma^\mu = (f+1)\Gamma_1^\mu - \Gamma_2^\mu f = \Gamma_2^\mu(f+1) - \Gamma_1^\mu f\,, \tag{A.15}$$

leads to a condition

$$(2f+1)\Gamma_1^\mu = \Gamma_2^\mu(2f+1)\,, \tag{A.16}$$

or, assuming existence of F^{-1}, to the relations

$$(2f+1)(1+a\Gamma\Phi)\gamma^\mu = \gamma^\mu(1+a\Gamma\Phi)(2f+1)\,, \tag{A.17}$$

which is satisfied with

$$f(\Gamma\Phi) = \frac{c - a\Gamma\Phi}{2(1+a\Gamma\Phi)}\,, \tag{A.18}$$

where c is an arbitrary number, which can be set to $c=0$. Using the solution (A.18) in the constraint (A.15) leads to an equation for F

$$\gamma^\mu = F\Big(\gamma^\mu + \frac{a}{2}\left\{\gamma^\mu\,,\Gamma\Phi\right\}\Big)F\,. \tag{A.19}$$

Assuming that the solution F of (A.19) exists, the transformed lagrangian (A.9) with the total derivative (A.13) added and with the function $f(\Gamma\Phi)$ given in (A.18) is

$$\begin{aligned}\mathcal{L} = &\frac{i}{2}\bar{\psi}'\gamma^\mu(\partial_\mu\psi') - \frac{i}{2}(\partial_\mu\bar{\psi}')\gamma^\mu\psi' - m\bar{\psi}'F(1+a\Gamma\Phi)F\psi' + g\bar{\psi}'F^2(\Gamma\Phi)\psi' \\ &+\frac{i}{2}\bar{\psi}'\Big(\gamma^\mu F^{-1}(\partial_\mu F) - (\partial_\mu F)F^{-1}\gamma^\mu + \frac{a}{2}F\left[\gamma^\mu\,,(\Gamma\partial_\mu\Phi)\right]F\Big)\psi' \\ &+\bar{\psi}'F\,\Lambda^\mu\,F\psi'\,A_\mu + \frac{a}{2}\bar{\psi}'F\,\left\{\Lambda_0^\mu\,,\Gamma\Phi\right\}\,F\psi'\,A_\mu\,.\end{aligned} \tag{A.20}$$

For scalar mesons $\Gamma\to\Gamma_s=1$ and hence Γ commutes with γ^μ. The matrix γ^μ can be factorized from (A.19), we get $F_s^2 = 1 + a_s\Phi_s$, and from (A.20) one immediately recovers (3.13).

For pseudoscalar mesons $\Gamma\to\Gamma_{ps} = -i\gamma^5$, and Γ anticommutes with γ^μ. Considering now $F\to F_{ps}(-i\gamma^5\Phi_{ps})$, for which $F_{ps}\gamma^\mu = \gamma^\mu F_{ps}^*$, we obtain from (A.19) the relation

$$F_{ps}F_{ps}^* = 1\,, \tag{A.21}$$

i.e., $F_{ps}^{-1} = F_{ps}^*$. Using these properties of F_{ps} we obtain from (A.20)

$$\begin{aligned}\mathcal{L} = &\mathcal{L}_{kin}^N(\psi') + \frac{\nu_{ps}g_{ps}}{2m}\bar{\psi}'\gamma^\mu\gamma^5F^*(\partial_\mu\Phi_{ps})F\psi' - ig_{ps}(1-\nu_{ps})\bar{\psi}'F_{ps}^2\gamma^5\Phi_{ps}\,\psi' \\ &+m\bar{\psi}'(1-F_{ps}^2)\psi' + \frac{i}{2}\bar{\psi}'\Big(F_{ps}\gamma^\mu(\partial_\mu F_{ps}) - (\partial_\mu F_{ps})\gamma^\mu F_{ps}\Big)\psi' \\ &+\bar{\psi}'F_{ps}\Lambda^\mu F_{ps}\,\psi' A_\mu - i\frac{a_{ps}}{2}\bar{\psi}'F_{ps}\left\{\Lambda_0^\mu, \gamma^5\Phi_{ps}\right\}F_{ps}\,\psi' A_\mu\,.\end{aligned} \tag{A.22}$$

The most general form of $F_{ps}(-i\gamma^5\Phi_{ps})$, satisfying the constraints (A.6), and (A.21) is

$$F_{ps}(-i\gamma^5\Phi_{ps}) = \exp\left(i\eta\frac{g_{ps}}{2m}\Phi_{ps}\gamma^5\,f(\Phi_{ps}^2)\right)\,, \tag{A.23}$$

where η is real number and $f^*(\Phi_{ps}^2) = f(\Phi_{ps}^2)$, normalized to $f(0)=1$. Up to the quadratic terms in Φ_{ps} the lagrangian (A.22) does not depend on the form of $f(\Phi_{ps}^2)$ and we can replace $f(\Phi_{ps}^2)\to f(0)=1$:

$$
\begin{aligned}
\mathcal{L} \simeq \mathcal{L}^N_{kin}(\psi') &+ \frac{(\nu_{ps}-\eta)g_{ps}}{2m}\bar{\psi}'\gamma^\mu\gamma^5(\partial_\mu\Phi_{ps})\psi' - ig_{ps}(1-\nu_{ps}+\eta)\bar{\psi}'\gamma^5\Phi_{ps}\psi' \\
&+\eta(1-\nu_{ps}+\frac{\eta}{2})\frac{g^2_{ps}}{m}\,\bar{\psi}'\Phi^2_{ps}\psi' + \eta(2\nu_{ps}-\eta)\frac{g^2_{ps}}{4m^2}\,\bar{\psi}'\gamma^\mu\vec{\tau}\cdot\vec{\Phi}_{ps}\times(\partial_\mu\vec{\Phi}_{ps})\,\psi' \\
&+\bar{\psi}'\Lambda^\mu\psi'\,A_\mu + i\frac{g_{ps}}{2m}\,\bar{\psi}'\left\{(\eta-\nu_{ps})\Lambda^\mu_0 + \eta\Delta\Lambda^\mu\,,\gamma^5\Phi_{ps}\right\}\psi'\,A_\mu\,. \qquad \text{(A.24)}
\end{aligned}
$$

The transformed lagrangian contains a $psNN$ vertex with the PS-PV mixing (the mixing parameter is $\nu_{ps}-\eta$) and the quadratic contact vertices of the standard form. The e.m. coupling with Λ^μ_0 is just the the usual Kroll-Rudermann coupling [15], it is obtained by the minimal coupling from the $psNN$ vertex with a derivative and it contains the same factor $(\nu_{ps}-\eta)$. For pseudoscalar mesons the interactions with derivatives of nucleon fields is replaced by the PV form of meson-nucleon coupling with derivative of the meson field. Thus, in this case transforming away the off-shell coupling means eliminating the PV $psNN$ vertex, which is achieved if one sets $\eta=\nu_{ps}$. The transformed lagrangian then simplifies to

$$
\begin{aligned}
\mathcal{L} \simeq \mathcal{L}^N_{kin}(\psi') &- ig_{ps}\bar{\psi}'\gamma^5\Phi_{ps}\psi' \\
&+\nu_{ps}(1-\frac{\nu_{ps}}{2})\frac{g^2_{ps}}{m}\,\bar{\psi}'\Phi^2_{ps}\psi' + \frac{g^2_{ps}\nu^2_{ps}}{4m^2}\,\bar{\psi}'\gamma^\mu\vec{\tau}\cdot\vec{\Phi}_{ps}\times(\partial_\mu\vec{\Phi}_{ps})\,\psi' \\
&+\bar{\psi}'\Lambda^\mu\psi'\,A_\mu + i\frac{\nu_{ps}g_{ps}}{2m}\,\bar{\psi}'\left\{\Delta\Lambda^\mu\,,\gamma^5\Phi_{ps}\right\}\psi'\,A_\mu\,. \qquad \text{(A.25)}
\end{aligned}
$$

For the vector mesons $\Phi\to v_\mu$, $\Gamma\to\gamma^\mu$ and the off-shell vertex in momentum space reads

$$
\begin{aligned}
\tilde{\Gamma}^\mu_v(p',p) &= \gamma^\mu + \frac{\nu_v}{2m}\left((\hat{p}'-m)\gamma^\mu + \gamma^\mu(\hat{p}-m)\right) \\
&= (1-\nu_v)\gamma^\mu + \frac{\nu_v}{2m}\left((p'+p)^\mu + i\sigma^{\mu\rho}(p'-p)_\rho\right)\,. \qquad \text{(A.26)}
\end{aligned}
$$

From (A.19) we get for the vector mesons

$$
\gamma^\mu = F_v(\gamma^\mu + a_v v^\mu)F\,. \qquad \text{(A.27)}
$$

Let us first consider isoscalar vector mesons. In this case, v_μ commutes with $\hat{v}$, and hence with $F_v(\hat{v})$. Therefore, we can multiply (A.27) by v_μ and find the solution

$$
F_v(\hat{v}) = \frac{1}{\sqrt{1+a_v\hat{v}}}\,, \qquad \text{(A.28)}
$$

which as a matrix in the Dirac space is defined by its Taylor series. The transformed lagrangian reads

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}^{kin}_N(\psi') + g_v\bar{\psi}'\frac{\hat{v}}{1+a_v\hat{v}}\psi' \\
&\quad+\frac{i}{2}\bar{\psi}'\Big(\gamma^\mu F_v^{-1}(\partial_\mu F_v) - (\partial_\mu F_v)F_v^{-1}\gamma^\mu + \frac{a_v}{2}F_v\left[\gamma^\mu\,,(\partial_\mu\hat{v})\right]F_v\Big)\psi' \\
&\quad+\bar{\psi}'F_v\,\Lambda^\mu\,F_v\psi'\,A_\mu + \frac{a_v}{2}\bar{\psi}'F_v\,\left\{\Lambda^\mu_0\,,\hat{v}\right\}\,F_v\psi'\,A_\mu \\
&\simeq \mathcal{L}^{kin}_N(\psi') + g_v\bar{\psi}'\hat{v}\psi' - \frac{a_v}{2}\bar{\psi}'\left\{\Delta\Lambda^\mu\,,\hat{v}\right\}\,\psi'\,A_\mu \\
&\quad-\frac{\nu_v g^2_v}{m}\bar{\psi}'(v_\rho v^\rho)\psi' - i\frac{\nu^2_v g^2_v}{8m^2}\bar{\psi}'\Big(\hat{v}\gamma^\mu(\partial_\mu\hat{v}) - (\partial_\mu\hat{v})\gamma^\mu\hat{v}\Big)\psi'\,. \qquad \text{(A.29)}
\end{aligned}
$$

For isovector vector mesons a solution to all order in meson fields does not seem to exist. The problem is that in this case $v_\alpha = \vec{\tau} \cdot \vec{v}_\alpha$ and the components of the vector field do not commute

$$[v_\alpha, v_\beta] = 2i\,\vec{\tau} \cdot (\vec{v}_\alpha \times \vec{v}_\beta)\,. \tag{A.30}$$

Still, if only terms linear in the meson field are retained, the choice $F_v \simeq 1 - a_v \hat{v}/2$ eliminates the interaction terms with derivatives of nucleon fields. Up to a quadratic order, the most general form of F_v is

$$F_v \simeq 1 - \frac{a_v}{2}\hat{v} + c a_v^2 \hat{v}\hat{v} + d a_v^2 v_\alpha \hat{v} \gamma^\alpha\,, \tag{A.31}$$

and it is easy to check that it does not solve (A.27) up to the quadratic order for any choice of c and d. This means that any field redefinition leaves some interaction terms quadratic in vector field and containing the derivatives of nucleon field.

To sum it up, the linear approximation to function F is in all cases (with $\eta = \nu_{ps}$ for pseudoscalar mesons) given by

$$F \simeq 1 - \frac{a}{2}\Gamma\Phi\,, \tag{A.32}$$

and the transformed lagrangian up to this order reads

$$\mathcal{L}(\psi') = \mathcal{L}_N^{kin}(\psi') + g\bar{\psi}'\Gamma\Phi\psi' - \frac{a}{2}\bar{\psi}'\,\{\Delta\Lambda^\mu\,, \Gamma\Phi\}\;\psi'\,A_\mu\,.. \tag{A.33}$$

For scalar, pseudoscalar and vector isoscalar mesons the transformation can be carried out to all orders and different contact interactions appear. For vector isovector mesons it seems impossible to transform away the interaction terms with derivatives of nucleon fields to higher than linear order in meson field. Although the closed solution exists for most types of mesons, if more than one off-shell vertex is considered one has to resort to approximate solution approximating F by a power series.

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